Math 3A Review of Sec 3.3 – 3.6 Summer 2024

Remember to show all work:

1. Find $\frac{dy}{dx}$ and simplify if possible:

A. $y=\csc^3(5x^3+2)$

B. $y=\sin^{-1}\left(\frac{4}{x^2}\right) = \sin^{-1}(4x^{-2})$

C. $y=\left(\frac{3x^2-5}{4x^2+3}\right)^5$

D. $y=(\ln x)^{\csc x}$

A) $\frac{dy}{dx} = 3\csc^2(5x^3+2)\left[-\csc(5x^3+2)\cot(5x^3+2)\right](15x^2)$

$$\boxed{\frac{dy}{dx} = -45x^2\csc^3(5x^3+2)\cot(5x^3+2)}$$

B) $\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{4}{x^2}\right)^2}} \cdot -8x^{-3} = \frac{1}{\sqrt{1-\frac{16}{x^4}}} \cdot \frac{-8}{x^3} = \frac{1}{\sqrt{\frac{x^4-16}{x^4}}} \cdot \frac{-8}{x^3}$

$\frac{dy}{dx} = \frac{1}{\frac{\sqrt{x^4-16}}{x^2}} \cdot \frac{-8}{x^3} = \boxed{\frac{-8}{x\sqrt{x^4-16}}}$

C) $y=\left(\frac{3x^2-5}{4x^2+3}\right)^5$ $\quad \frac{dy}{dx} = 5\left(\frac{3x^2-5}{4x^2+3}\right)^4 \cdot \frac{6x(4x^2+3)-8x(3x^2-5)}{(4x^2+3)^2}$

$\frac{dy}{dx} = \frac{5(3x^2-5)^4}{(4x^2+3)^4} \cdot \frac{\cancel{24x^3}+18x-\cancel{24x^3}+40x}{(4x^2+3)^2} = \boxed{\frac{290x(3x^2-5)^4}{(4x^2+3)^6}}$

D) $\ln y = \ln(\ln x)^{\csc x}$

$\ln y = \csc x(\ln(\ln x))$

$\frac{1}{y}\cdot\frac{dy}{dx} = -\csc x\cot x \cdot \ln(\ln x) + \csc x \cdot \frac{1}{\ln x}\cdot\frac{1}{x})$

$y\left(\frac{1}{y}\cdot\frac{dy}{dx}\right) = \left[-\csc x\cot x\cdot\ln(\ln x) + \frac{\csc x}{x\ln x}\right]y$

$$\boxed{\frac{dy}{dx} = \left[-\csc x\cot x\cdot\ln(\ln x) + \frac{\csc x}{x\ln x}\right](\ln x)^{\csc x}}$$

2. Evaluate the limit: $\lim_{\Theta \to 0} \frac{\cos\Theta - 1}{2\Theta^2} \cdot \frac{\cos\theta + 1}{\cos\theta + 1}$

$$\lim_{\theta \to 0} \frac{\cos^2\theta - 1}{2\theta^2(\cos\theta + 1)} = \lim_{\theta \to 0} \frac{-\sin^2\theta}{2\theta^2(\cos\theta + 1)}$$

$$\lim_{\theta \to 0} -\frac{1}{2} \cdot \left(\frac{\sin\theta}{\theta}\right)^2 \cdot \frac{1}{\cos\theta + 1}$$

$$-\frac{1}{2} \cdot 1 \cdot \frac{1}{2}$$

$$\boxed{-\frac{1}{4}}$$

3. Find the equation of the tangent line to the curve at the given value: $y = \sqrt{1 + x^3}$ at $x = 2$

$$y = (1 + x^3)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}(1 + x^3)^{-1/2}(3x^2)$$

$$\frac{dy}{dx} = \frac{3x^2}{2\sqrt{1 + x^3}}$$

at $x = 2$

$$y = \sqrt{1 + 2^3} = 3$$

$$\frac{dy}{dx} = \frac{3(2)^2}{2\sqrt{1 + 2^3}} = \frac{12}{6} = 2$$

$(2, 3)$ $m = 2$

$$y - 3 = 2(x - 2)$$

$$y - 3 = 2x - 4$$

$$\boxed{y = 2x - 1}$$

4. Find the derivative using properties of logarithms: $y = \ln\left(\frac{x^7 \tan(4x)}{\sqrt[3]{4x^2+3}}\right)$

$$y = \ln x^7 + \ln(\tan 4x) - \ln(4x^2+3)^{1/3}$$

$$y = 7\ln x + \ln(\tan 4x) - \frac{1}{3}\ln(4x^2+3)$$

$$\frac{dy}{dx} = 7 \cdot \frac{1}{x} + \frac{1}{\tan 4x} \cdot \sec^2(4x) \cdot 4 - \frac{1}{3} \cdot \frac{1}{4x^2+3} \cdot 8x$$

$$\boxed{\frac{dy}{dx} = \frac{7}{x} + \frac{4\sec^2(4x)}{\tan 4x} - \frac{8x}{3(4x^2+3)}}$$