Math 3A Practice Problems #2 Summer 2024

Remember to show all work!!

1. Find the equation of the tangent line to the curve $y=\frac{-5}{x+2}$ at $x=1$. (Use the limit definition of a derivative.)

$$f'(x)=\lim_{h\to 0}\left(\frac{\frac{-5}{x+h+2}-\frac{-5}{x+2}}{h}\right)\frac{(x+h+2)(x+2)}{(x+h+2)(x+2)}=\lim_{h\to 0}\frac{-5(x+2)+5(x+h+2)}{h(x+h+2)(x+2)}$$

$$\lim_{h\to 0}\frac{-5x-10+5x+5h+10}{h(x+h+2)(x+2)}=\lim_{h\to 0}\frac{5h}{h(x+h+2)(x+2)}=\frac{5}{(x+2)^2}$$

$$\left(1,-\frac{5}{3}\right)\quad m=\frac{5}{3^2}=\frac{5}{9}$$

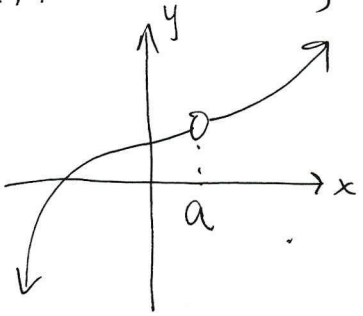
$$y+\frac{5}{3}=\frac{5}{9}(x-1)$$

$$y+\frac{15}{9}=\frac{5}{9}x-\frac{5}{9}$$

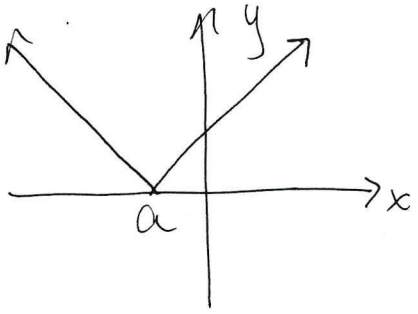
$$\boxed{y=\frac{5}{9}x-\frac{20}{9}}$$

2. What are the three ways a function is not differentiable at a point $x=a$? (Sketch a graph for each possibility)

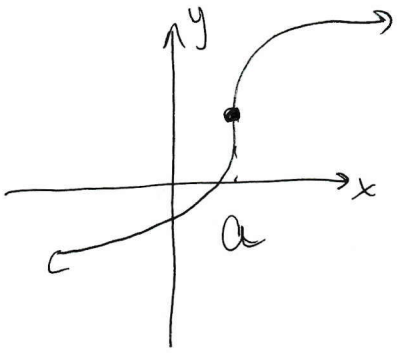
A) A discontinuity



B) A sharp point (cusp)



C) A vertical tangent line



3. Find the equations for two lines that are both tangent to the curve $y = x^3 - 3x^2 + 3x - 3$ and parallel to the line $3x - y = 15$.

$3x - y = 15$

$-y = -3x + 15$

$y = 3x - 15$

parallel lines have the same slope

so $m = 3$

$\frac{dy}{dx} = 3x^2 - 6x + 3$

the derivative represents the slope of the tangent lines

$3 = 3x^2 - 6x + 3$

$0 = 3x^2 - 6x$

$0 = 3x(x-2)$

$x = 0, \quad x = 2$

Two points on the curve with tangent lines of slope 3

$y = x^3 - 3x^2 + 3x - 3$ $(0, -3)$ and $(2, -1)$

$(0, -3)$ $\boxed{y = 3x - 3}$

$(2, -1)$ $y + 1 = 3(x - 2)$

$y + 1 = 3x - 6$

$\boxed{y = 3x - 7}$

4. Find all values of x where the function $f(x) = \frac{x^2 - 1}{x + 2}$ has a horizontal tangent line.

Horizontal lines have slope 0

$$f'(x) = \frac{2x(x+2) - (x^2-1)(1)}{(x+2)^2} = \frac{2x^2 + 4x - x^2 + 1}{(x+2)^2} = \frac{x^2 + 4x + 1}{(x+1)^2}$$

$$\frac{x^2 + 4x + 1}{(x+1)^2} = 0 \quad , \quad x^2 + 4x + 1 = 0$$

$$x = \frac{-4 \pm \sqrt{16 - 4(1)(1)}}{2(1)} = \frac{-4 \pm \sqrt{12}}{2} = \frac{-4 \pm 2\sqrt{3}}{2}$$

$$\boxed{x = -2 \pm \sqrt{3}}$$

5. Find and simplify the derivative:

A. $y = \dfrac{3x^2+5}{7x^2-3}$

B. $y = \dfrac{4x^2-3x+7}{\sqrt[3]{x}}$

A) $\dfrac{dy}{dx} = \dfrac{6x(7x^2-3)-(3x^2+5)(14x)}{(7x^2-3)^2} = \dfrac{42x^3-18x-42x^3-70x}{(7x^2-3)^2} = \boxed{\dfrac{-88x}{(7x^2-3)^2}}$

B) $y = \dfrac{4x^2}{x^{1/3}} - \dfrac{3x}{x^{1/3}} + \dfrac{7}{x^{1/3}} = 4x^{5/3} - 3x^{2/3} + 7x^{-1/3}$

$\boxed{\dfrac{dy}{dx} = \dfrac{20}{3}x^{2/3} - 2x^{-1/3} - \dfrac{7}{3}x^{-4/3}}$

OR

$\boxed{\dfrac{dy}{dx} = \dfrac{20}{3}x^{2/3} - \dfrac{2}{x^{1/3}} - \dfrac{7}{3x^{4/3}}}$